## Econ102 seminar questions

Week 8 (relates to material in Week 7 lectures)
Game theory

# Note that: This is just brief solutions. To understand the answer of each question, revise what we discussed in the tutorial. (Let me know if you see any error) 

1. In studying for his Economics final, Sam is concerned about only two things: his grade and the amount of time he spends studying. A good grade will give him a benefit of 20; an average grade, a benefit of 5 ; and a poor grade, a benefit of 0 . By studying a lot, Sam will incur a cost of 10; by studying a little, a cost of 6 . Moreover, if Sam studies a lot and all other students study a little, he will get a good grade and they will get poor ones. But if they study a lot and he studies a little, they will get good grades and he will get a poor one. Finally, if he and all other students study for the same amount of time, everyone will get average grades. Other students share Sam's preferences regarding grades and study time.
a. Model this situation as a two-person prisoner's dilemma in which the strategies are to study a little and to study a lot, and the players are Sam and all other students. Include the payoffs in the matrix.
b. What is the equilibrium outcome in this game? From the students' perspective, is it the best outcome?

The payoff matrix is (payoffs are All others, Sam):

| Sam | Study a lot | All others |  |
| :---: | :---: | :---: | :---: |
|  |  | Study a lot | Study a little |
|  |  | $(-5,-5)$ | $(-6,10)$ |
|  | Study a little | $(10,-6)$ | $(-1,-1)$ |

The equilibrium outcome is that all study a lot and receive an average grade. From the students' perspective, for everyone to study a little would have been better.
2. Consider the following "dating game", which has two players, $A$ and $B$, and two strategies, to buy a cinema ticket or a football ticket. The payoffs (A, B), given in points, are as shown in the matrix below.

B
Buy cinema ticket
Buy football ticket
A

| Buy cinema ticket | Buy football ticket |
| :--- | :--- |
| $(2,3)$ | $(0,0)$ |
| $(1,1)$ | $(3,2)$ |

Assume that $A$ and $B$ buy their tickets separately and simultaneously. Each must decide what to do knowing the available choices and payoffs but not what the other has actually chosen. Each player believes the other to be rational and self-interested.
a. Does either player have a dominant strategy?
b. How many potential equilibria are there?
c. Is the game a prisoner's dilemma? Explain.
d. Suppose player A gets to buy her ticket first. Player B does not observe A's choice but knows that A chose first. Player A knows that player B knows she chose first. What is the equilibrium outcome?
e. Suppose the situation is similar to part (d), except that player B chooses first. What is the equilibrium outcome?

There is no dominant strategy. The best choice for each player depends on what the other player does.

The top-left and bottom-right cells are both potential equilibria. In each of those cells, neither player has any incentive to change strategies.

The payoffs do not follow the pattern associated with a prisoner's dilemma, because neither player has a dominant strategy.

A knows that if he has the first move and buys a cinema ticket, so will $B$, in which case $A$ will get a payoff of 2. If A buys a football ticket, so will B, in which case A will get a payoff of 3 . So A will buy a football ticket, and so will B. (Revise the discussion in the tutorial for details)

If $B$ has the first move, they will both see a movie. (Revise the discussion in the tutorial for details)
3. Blackadder and Baldrick are rational, self-interested criminals imprisoned in separate cells in a dark medieval dungeon. They face the prisoner's dilemma displayed in the matrix below (payoffs are Blackadder, Baldrick).

Baldrick

$$
\begin{aligned}
& \text { Confess } \\
& \text { Deny }
\end{aligned}
$$

| Blackadder |
| :--- |
| Confess |


| $(-5,-5)$ | $(-20,0)$ |
| :--- | :--- |
| $(0,-20)$ | $(-1,-1)$ |

Assume that Blackadder is willing to pay $£ 1,000$ for each year by which he can reduce his sentence below 20 years. A corrupt jailer tells Blackadder that before he decides whether to confess or deny the crime, he can tell him Baldrick's decision. How much is this information worth to Blackadder?

The information is worth nothing to Blackadder, who knows that Baldrick's dominant strategy is to confess. In any case, Blackadder also has a dominant strategy of his own (also to confess).
4. You are sitting in your car in a university car park that is currently full, waiting for a place to become free. Just as one becomes free a driver who has just arrived overtakes you in order to park in the vacated spot before you can. Suppose this driver would be willing to pay up to $£ 10$ to park in that spot, and up to $£ 30$ to avoid getting into an argument with you. At the same time the other driver guesses, accurately, that you too would be willing to pay up to $£ 30$ to avoid a confrontation, and up to $£ 10$ to park in the vacant spot.
a. Model this situation as a two-stage decision tree in which the other driver's bid to take the space is the opening move and your strategies are (1) to protest and (2) not to protest. If you protest (initiate an argument), the rules of the game specify that the other driver has to let you take the space. Show the payoffs at the end of each branch of the tree.
b. What is the equilibrium outcome?

## (Revise the discussion in the tutorial for details)

The decision tree is:


The top branch at $A$ is unattractive to the other driver. Since you get a higher payoff on the bottom branch at $B$, and the other driver knows it, the equilibrium outcome is that he gets the space and you keep waiting.
5. The owner of a thriving business wants to hire someone who will manage a new office in a distant city honestly. He can afford to pay a weekly salary of $£ 2,000$ ( $£ 1,000$ more than the manager would be able to earn elsewhere), and still earn an economic profit of $£ 800$. He fears that he will not be able to monitor the behaviour of the manager, who will therefore be in a position to embezzle money from the business. The owner knows that if the remote office is managed dishonestly, the manager can "earn" $£ 3,100$ while causing the owner an economic loss of $£ 600$ per week.
a. If the owner believes that all managers are narrowly self-interested income maximisers, will he open the new office?
b. Suppose the owner knows that a managerial candidate is a devoutly religious person who condemns dishonest behaviour and who would be willing to pay up to $£ 15,000$ to avoid the guilt she would feel if she were dishonest. Will the owner open the remote office?

The owner knows that if he opens the remote office (top branch at A), the potential manger's best strategy is to be dishonest (bottom branch at B), in which case the owner will get $-£ 600$. Since the owner gets nothing by choosing the bottom branch at $A$, he will not open the new office.


If the manager is devoutly religious, then the owner will open the remote office. This time, the potential manager's payoff on the bottom branch at $B$ is $£ 3,100-£ 15,000=-£ 11,900$, so the owner knows the manager will choose the top branch at B.
6. You are playing a game of "matching pennies" with a friend. Each of you has a penny hidden in your hand, facing either heads up or tails up (you know which way the one in your hand is facing). On the count of "three" you simultaneously show your pennies to each other. If the face-up side of your coin matches the face-up side of your friend's coin, you get to keep the two pennies. If the faces do not match, your friend gets to keep the pennies.
a. Who are the players in this game? What are each player's strategies? Construct a payoff matrix for the game.
b. Is there a dominant strategy? If so, what?
c. Is there an equilibrium? If so, what?

The players are you and your friend. Your strategy choices are heads or tails. The following matrix describes the payoffs (you, your friend), measured as the change in the number of pennies each player owns.

| Your friend |  | You |  |
| :---: | :---: | :---: | :---: |
|  |  | Heads | Tails |
|  | Heads | $(1,-1)$ | (-1, 1) |
|  | Tails | $(-1,1)$ | $(1,-1)$ |

There are no dominant strategies and there is no equilibrium, because if your friend plays one side, you want to match that side. But if you match, your friend will want to change strategies.

